

$p\bar{p} \rightarrow \Lambda_c \bar{\Lambda}_c$ within the generalized parton picture - first results

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We study proton-antiproton annihilation into $\Lambda_c \bar{\Lambda}_c$ pairs within the generalized parton picture. Our starting point is the double handbag diagram which is shown to factorize into soft generalized parton distributions for the $p \rightarrow \Lambda_c$ (and $\bar{p} \rightarrow \bar{\Lambda}_c$) transition and a hard subprocess amplitude for $u\bar{u} \rightarrow c\bar{c}$. Thereby the mass of the charm quark is taken as the hard scale so that our results are not restricted to large scattering angles and/or incredibly large energies. Modelling the generalized parton distributions for the $p \rightarrow \Lambda_c$ transition by an overlap of simple quark-diquark light-cone wave functions we make first predictions for $p \rightarrow \Lambda_c$ transition form factors and unpolarized $p\bar{p} \rightarrow \Lambda_c \bar{\Lambda}_c$ cross sections. Our findings may become interesting in view of forthcoming experiments at FAIR in Darmstadt.

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1. Introduction

Exclusive hadronic reactions which require the production of heavy quark-antiquark pairs are, to a large extent, cleaner and easier to handle than those in which only light flavors are involved. On the one hand, certain elementary reaction mechanisms can already be ruled out from the beginning due to the small heavy-flavor content of the quark sea. On the other hand, the mass of the heavy quark itself can serve as a hard scale so that there is a good chance that QCD perturbation theory provides a substantial part of the process amplitude already at moderately large energies.

The simplest elementary reaction mechanism for the process we are interested in, namely $p\bar{p} \rightarrow \Lambda_c \bar{\Lambda}_c$, is depicted in Fig. 1. This is the mechanism which we assume to be dominant in the forward hemisphere for energies well above production threshold. We are going to analyze it in terms of generalized parton distributions [1],[2].

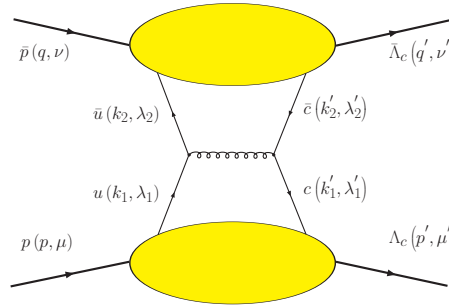


Figure 1: Double-handbag contribution to $p\bar{p} \rightarrow \Lambda_c \bar{\Lambda}_c$.

2. Kinematics

We denote particle momenta and helicities as shown in Fig. 1. For our purposes it is most convenient to work in a center-of-mass frame, in which the light-cone (LC) components of the proton and Λ_c momenta are parameterized as ¹

$$p = \left[(1 + \xi) \bar{p}^+, \frac{m^2 + \Delta_\perp^2/4}{2(1 + \xi) \bar{p}^+}, -\frac{\Delta_\perp}{2} \right] \quad \text{and} \quad p' = \left[(1 - \xi) \bar{p}^+, \frac{M^2 + \Delta_\perp^2/4}{2(1 - \xi) \bar{p}^+}, \frac{\Delta_\perp}{2} \right], \quad (2.1)$$

respectively. M stands for the Λ_c mass and m for the mass of the proton. The corresponding antiparticle momenta are $q = [p^-, p^+, \Delta_\perp/2]$ and $q' = [p'^-, p'^+, -\Delta_\perp/2]$. The average of proton and Λ_c momenta $\bar{p} = \frac{1}{2}(p + p')$ defines the longitudinal direction and the 4-momentum transfer is specified by $\Delta = p' - p = q - q'$. The relative momentum transfer in longitudinal direction is given by the “skewness parameter”

$$\xi \equiv \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2\bar{p}^+}. \quad (2.2)$$

¹We use the following convention for the LC components of a 4-vector v : $v = [v^+, v^-, v_\perp]$ with $v^\pm = 1/\sqrt{2}(v^0 \pm v^3)$ and $v_\perp = (v^1, v^2)$.

3. Factorization

If the dynamical mechanism underlying $p\bar{p} \rightarrow \Lambda_c \bar{\Lambda}_c$ scattering is given by Fig. 1, the corresponding scattering amplitude can be written as ($k_i^{\text{av}} = (k_i + k'_i)/2$)

$$M_{\mu' \nu', \mu \nu} = \int d^4 k_1^{\text{av}} \theta(k_1^{\text{av}+}) \int \frac{d^4 z_1}{(2\pi)^4} e^{i k_1^{\text{av}} z_1} \int d^4 k_2^{\text{av}} \theta(k_2^{\text{av}-}) \int \frac{d^4 z_2}{(2\pi)^4} e^{i k_2^{\text{av}} z_2} H(k'_1, k'_2; k_1, k_2) \\ \times \langle \Lambda_c : p', \mu' | T \bar{\Psi}^c \left(-\frac{z_1}{2} \right) \Psi^u \left(\frac{z_1}{2} \right) | p : p, \mu \rangle \langle \bar{\Lambda}_c : q', \nu' | T \bar{\Psi}^u \left(\frac{z_2}{2} \right) \Psi^c \left(-\frac{z_2}{2} \right) | \bar{p} : q, \nu \rangle, \quad (3.1)$$

with $H(k'_1, k'_2; k_1, k_2)$ representing the scattering amplitude for the elementary subprocess

$$u(k_1, \lambda_1) \bar{u}(k_2, \lambda_2) \rightarrow c(k'_1, \lambda'_1) \bar{c}(k'_2, \lambda'_2). \quad (3.2)$$

The hadronic matrix elements describe the emission of a light (anti)quark by the (anti)proton and the absorption of a charm (anti)quark by the (anti) Λ_c . For better readability we have suppressed helicity labels as well as color and spinor indices (and corresponding sums) for the quarks.

In order to simplify the right-hand-side of Eq. (3.1) we make a few plausible assumptions. In analogy to Compton scattering [3], [4] we assume that the process is dominated by partons with restricted virtualities and transverse momenta. To be more precise, we demand for the active (anti)quarks that $k_i^2 \lesssim \Lambda^2$ and $|k_i'^2 - m_c^2| \lesssim \Lambda^2$ (cf. Fig. 1 for the assignment of (anti)quark momenta). The relative transverse momentum components should satisfy ²

$$\tilde{k}_{\perp i}^2/x_i \lesssim \Lambda^2, \quad \hat{k}_{\perp i}^{\prime 2}/x'_i \lesssim \Lambda^2, \quad (3.3)$$

with Λ being a hadronic scale of the order of 1 GeV and m_c the charm-quark mass. The restrictions for the transverse momenta are supposed to hold in the hadron in/out frames (tilde/hat), in which the corresponding parent hadrons carry no transverse momenta. Analogous constraints should also hold for spectator partons. Furthermore, the hadronic matrix elements represented by the blobs in Fig. 1 are assumed to exhibit a pronounced peak at $x_0 = m_c/M \approx 0.6 - 0.7$. This assumption reflects the expected behavior of light-cone wave functions for the Λ_c and is also supported by similar observations for heavy-quark fragmentation functions.

Since the virtuality of the gluon in the subprocess amplitude has to be at least $4m_c^2$, it occurs to be natural to take the charm-quark mass m_c as a hard scale. Under the foregoing assumptions on the parton momenta it is then a good approximation to neglect the relative transverse momenta ($\tilde{k}_{\perp i}$ and $\hat{k}_{\perp i}'$) of the (anti)quarks in the subprocess amplitude $H(k'_1, k'_2; k_1, k_2)$ and to replace the momenta of the active quarks $k_i^{(\prime)}$ by (on-shell) momenta $\bar{k}_i^{(\prime)}$ which are collinear to those of the corresponding hadrons. With these approximations in the subprocess amplitude most of the integrations in Eq. (3.1) can be done analytically. The resulting delta functions enforce a light-like separation of the fields in the hadronic matrix elements. This allows us to drop the time ordering of the fields. Due to our assumption on the hadronic matrix elements it is also justified to apply a

²The plus-components of parton momentum k and hadron momentum p are related by $k^+ = xp^+$. For further purposes it is also convenient to introduce an “average” momentum fraction of the active quarks, i.e. $\bar{x}_1 = (k_1^{\text{av}+})/(\bar{p}^+)$ (and analogously for the active antiquarks)

“peaking approximation” to the subprocess amplitude (i.e. replace all x_i by x_0). In this way we are able to write the hadronic amplitude as a product of integrals over soft hadronic matrix elements with a hard partonic subprocess amplitude:

$$\begin{aligned} M_{\mu' \nu', \mu \nu}(p', q'; p, q) &= H(\bar{k}'_1, \bar{k}'_2; \bar{k}_1, \bar{k}_2) \big|_{x_i^{(q)}=x_0} \\ &\times \int dk_1^{\text{av}+} \theta(k_1^{\text{av}+}) \int \frac{dz_1^-}{2\pi} e^{ik_1^{\text{av}+} z_1^-} \langle \Lambda_c : p', \mu' | \bar{\Psi}^c \left(-\frac{z_1^-}{2} \right) \Psi^u \left(\frac{z_1^-}{2} \right) | p : p, \mu \rangle \\ &\times \int dk_2^{\text{av}-} \theta(k_2^{\text{av}-}) \int \frac{dz_2^+}{2\pi} e^{ik_2^{\text{av}-} z_2^+} \langle \bar{\Lambda}_c : q', \nu' | \bar{\Psi}^u \left(\frac{z_2^+}{2} \right) \Psi^c \left(-\frac{z_2^+}{2} \right) | \bar{p} : q, \nu \rangle. \end{aligned} \quad (3.4)$$

The subprocess amplitude can be calculated perturbatively, whereas the hadronic matrix elements describe the non-perturbative transitions of the (anti-)proton to the (anti-) Λ_c by emission of a (anti) u quark and reabsorption of a (anti) c quark. It are just the hadronic matrix elements which give rise to generalized parton distribution functions (GPDs) and to form factors, which are essentially $1/x$ moments of the GPDs [1], [2]. Therefore we will have a closer look on them.

After some algebraic manipulations (insertion of energy- and helicity-projectors, etc.) the first double integral in Eq. (3.4) can be split up into three (independent) terms of the form

$$\int_{\xi}^1 \frac{d\bar{x}_1}{\sqrt{\bar{x}_1^2 - \xi^2}} \bar{p}^+ \int \frac{dz_1^-}{2\pi} e^{i\bar{x}_1 \bar{p}^+ z_1^-} \langle \Lambda_c : p' \mu' | \bar{\Psi}^c \Gamma \Psi^u | p : p\mu \rangle =: \int_{\xi}^1 \frac{d\bar{x}_1}{\sqrt{\bar{x}_1^2 - \xi^2}} \mathcal{H}_{\mu' \mu}^{cu}(\Gamma), \quad (3.5)$$

where $\Gamma = \gamma^+, \gamma^+ \gamma_5$, or $i\sigma^{+j}$. γ^+ and $\gamma^+ \gamma_5$ demand for the same helicity of the emitted and absorbed parton, whereas $i\sigma^{+j}$ is associated with a helicity flip. At this point the GPDs are introduced by decomposing $\mathcal{H}_{\mu' \mu}^{cu}(\Gamma)$ into its covariant structures:

$$\mathcal{H}_{\mu' \mu}^{cu}(\gamma^+) = \bar{u}(p', \mu') \left[H^{cu}(\bar{x}, \xi, t) \gamma^+ + E^{cu}(\bar{x}, \xi, t) \frac{i\sigma^{+\nu} \Delta_\nu}{M+m} \right] u(p, \mu), \quad (3.6)$$

$$\mathcal{H}_{\mu' \mu}^{cu}(\gamma^+ \gamma_5) = \bar{u}(p', \mu') \left[\tilde{H}^{cu}(\bar{x}, \xi, t) \gamma^+ \gamma_5 + \tilde{E}^{cu}(\bar{x}, \xi, t) \frac{\gamma_5 \Delta^+}{M+m} \right] u(p, \mu), \quad (3.7)$$

$$\begin{aligned} \mathcal{H}_{j\mu' \mu}^{Tcu}(i\sigma^{+j}) &= \bar{u}(p', \mu') \left[H_T^{cu}(\bar{x}, \xi, t) i\sigma^{+j} + \tilde{H}_T^{cu}(\bar{x}, \xi, t) \frac{\bar{p}^+ \Delta^j - \Delta^+ \bar{p}^j}{Mm} + \right. \\ &\quad \left. E_T^{cu}(\bar{x}, \xi, t) \frac{\gamma^+ \Delta^j - \Delta^+ \gamma^j}{M+m} + \tilde{E}_T^{cu}(\bar{x}, \xi, t) \frac{\gamma^+ \bar{p}^j - \bar{p}^+ \gamma^j}{(M+m)/2} \right] u(p, \mu). \end{aligned} \quad (3.8)$$

Analogous results hold for the hadronic matrix elements of the antiparticles.

What enters the scattering amplitude are rather moments of the GPDs than the GPDs themselves. It is thus convenient to introduce form factors

$$R_i(\xi, t) = \int_{\xi}^1 \frac{d\bar{x}_1}{\sqrt{\bar{x}_1^2 - \xi^2}} F_i(\bar{x}, \xi, t), \quad (3.9)$$

where F_i stands for any of the transition GPDs introduced in Eqs. (3.6) – (3.8). In our notation the form factors $R_V, R_T, R_A, R_P, S_T, S_{V1}, S_S, S_{V2}$ correspond to the GPDs $H, E, \tilde{H}, \tilde{E}, H_T, E_T, \tilde{H}_T, \tilde{E}_T$, respectively.

Combining Eqs. (3.5) – (3.9) and inserting them into Eq. (3.4) allows us to write the hadronic scattering amplitude in terms of these, a priori unknown, form factors and the hard subprocess amplitude. The result is a rather lengthy expression. Fortunately, some of the form factors can be neglected. By taking different helicity combinations for the matrix elements in Eqs. (3.6) – (3.8) we observe that all the form factors apart of R_V, R_A and S_T (corresponding to the GPDs H, \tilde{H} and H_T) must vanish in forward direction, since they are connected with orbital angular momentum. Also for large Mandelstam t these form factors should be suppressed (as compared to R_V, R_A and S_T) by powers of $\sqrt{-t}$, depending on how many units of angular momentum are involved. We therefore expect the following hierarchy of the form factors:

$$|R_V|, |R_A|, |S_T| \gg |R_T|, |R_P|, |S_S|, |S_{V1}|, |S_{V2}|. \quad (3.10)$$

In $M_{\mu' \nu', \mu \nu}(p', q'; p, q)$ a further suppression of contributions coming from helicity flip form factors is caused by the corresponding subprocess amplitudes. We therefore neglect all helicity-flip form factors apart of S_T . This provides us with comparably simple expressions for the $p\bar{p} \rightarrow \Lambda_c \bar{\Lambda}_c$ helicity amplitudes in terms of the form factors R_V, R_A and S_T and the subprocess amplitudes:

$$M_{+\pm, +\pm} = \frac{C_F}{2N_C} (1 - \xi^2) H_{+-, +-} (R_V^2 \mp R_A^2), \quad (3.11)$$

$$M_{--, ++} = 0, \quad (3.12)$$

$$M_{-+, +-} = \frac{C_F}{N_C} (1 - \xi^2) H_{-+, +-} S_T^2, \quad (3.13)$$

$$M_{++, \pm\mp} = \frac{C_F}{2N_C} (1 - \xi^2) H_{++, +-} S_T (R_V + R_A), \quad (3.14)$$

$$M_{\mp\pm, ++} = \mp \frac{C_F}{2N_C} (1 - \xi^2) H_{++, +-} S_T (R_V - R_A), \quad (3.15)$$

with

$$H_{+-, +-} = -8\pi \alpha_s (x_0^2 s) \cos^2(\theta_{\text{cm}}/2), \quad H_{-+, +-} = 8\pi \alpha_s (x_0^2 s) \sin^2(\theta_{\text{cm}}/2), \quad (3.16)$$

$$H_{++, +-} = -8\pi \alpha_s (x_0^2 s) \frac{M}{\sqrt{s}} \sin \theta_{\text{cm}}, \quad H_{++, -+} = H_{++, +-}. \quad (3.17)$$

$C_F = 4/3$ denotes the color factor, $N_C = 3$ is the number of colors. The remaining helicity amplitudes are related by parity invariance,

$$M_{-\mu' - \nu', -\mu - \nu} = (-1)^{\mu' - \nu' - \mu + \nu} M_{\mu' \nu', \mu \nu}. \quad (3.18)$$

For a more detailed account of these results and their derivation we refer to Ref. [5].

4. Modelling the GPDs

The kinematical requirement for the production of a $c\bar{c}$ pair $x \gtrsim 2x_0 M/\sqrt{s}$ implies that $x > \xi$.³ We are thus in the DGLAP region where GPDs can be modelled as overlap of light-cone wave functions of the proton and the Λ_c [6]. For the Λ_c it is certainly a good approximation to consider

³This is also the reason why the integral in Eq. (3.9) starts from ξ and not from 0.

only its valence Fock state. For the proton higher Fock states may be important, but the required overlap with the Λ_c projects out only appropriate spin-flavor combinations of its valence Fock state.

As a first attempt we consider a simple quark-diquark model for the baryons. We assume that a baryon consists of an active quark, which undergoes the hard scattering, and a diquark, which acts as a spectator. For the Λ_c the diquark has to be a spin-isospin scalar. In Ref. [7] $p\bar{p} \rightarrow \Lambda_c \bar{\Lambda}_c$ has already been studied within a quark-diquark model, but without using the general framework of GPDs. For our present investigation we will take a quark-diquark wave function for the Λ_c which is similar to the one used for a calculation of heavy-baryon transition form factors [8]:

$$|\Lambda_c^+ : \pm\rangle = |c_\pm S_{[ud]}\rangle \quad \text{with} \quad \Psi_\Lambda(x, k_\perp) = N_\Lambda (1-x) e^{-\frac{a_\Lambda^2}{x(1-x)} k_\perp^2} e^{-a_\Lambda^2 M^2 \frac{(x-x_0)^2}{x(1-x)}}. \quad (4.1)$$

x denotes the momentum fraction carried by the quark. The quark-diquark wave function of the proton, on the other hand, is chosen similar to the one in Ref. [7]:

$$|p : \pm\rangle(x, k_\perp) = |u_\pm S_{[ud]}\rangle \quad \text{with} \quad \Psi_p(x, k_\perp) = N_p (1-x) e^{-\frac{a_p^2}{x(1-x)} k_\perp^2}. \quad (4.2)$$

These are pure s -wave wave functions and hence all GPDs apart of H , \tilde{H} and H_T will vanish. Within this model the quark should have the same helicity as the baryon, which implies $\tilde{H} = H$. Likewise, also H_T is non-zero and we even have

$$H_T = \tilde{H} = H, \quad (4.3)$$

i.e. we are left with a single GPD. For the model wave functions (4.1) and (4.2) the overlap integral can be done analytically with the result

$$H(\bar{x}, \xi, \Delta_\perp^2) = \left(\frac{N_\Lambda N_p}{16\pi^2} \right) \frac{1}{(1-\xi)^{3/2}} \frac{(1-\bar{x})^3 (\bar{x}^2 - \xi^2)}{a_\Lambda^2 (1-\xi)^2 (\bar{x} + \xi) + a_p^2 (1+\xi)^2 (\bar{x} - \xi)} \\ \times \exp \left[-a_\Lambda^2 M^2 \frac{(\bar{x} - \xi - x_0(1-\xi))^2}{(\bar{x} - \xi)(1-\bar{x})} \right] \exp \left[-\Delta_\perp^2 \frac{a_\Lambda^2 a_p^2 (1-\bar{x})}{a_\Lambda^2 (1-\xi)^2 (\bar{x} + \xi) + a_p^2 (1+\xi)^2 (\bar{x} - \xi)} \right]. \quad (4.4)$$

With this GPD we can now evaluate the form factor $R \equiv R_V = R_A = S_T$.

5. Numerical Results and Conclusions

For our numerical calculations the transverse size parameters $a_\Lambda = a_p = 1.1 \text{ GeV}^{-1}$ are chosen in such a way that they provide a value of $\approx 300 \text{ MeV}$ for the mean intrinsic transverse momentum $\langle k_\perp^2 \rangle^{1/2}$ of a quark inside a baryon. The wave function normalizations are fixed such that the probabilities to find the quark-diquark states in a proton or a Λ_c are 0.5 and 0.9, respectively. The x -dependence for the resulting transition GPD H at different values of t (i.e. different values of the skewness parameter ξ) is shown in Fig. 2. Shown in the same figure is also the corresponding (scaled) transition form factor R as a function of t . With this form factor the unpolarized differential and integrated cross sections for $p\bar{p} \rightarrow \Lambda_c \bar{\Lambda}_c$ are easily evaluated. Corresponding predictions are plotted in Fig. 3. The integrated cross section is of the order of nb, i.e. comparable in magnitude with the result in Ref. [7]. This is still in the range of high precision experiments. Whether predictions from more realistic three-quark wave functions will be comparable to the outcome of this simple quark-diquark model is presently under investigation [5].

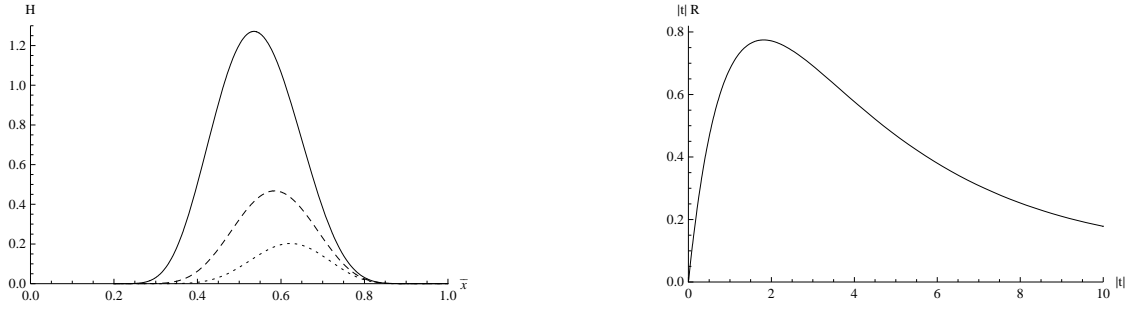


Figure 2: *Left:* The transition GPD H in the quark-diquark model versus \bar{x} at Mandelstam $s = 30 \text{ GeV}^2$ for $\Delta_\perp^2 = 0$ (solid), 2 (dashed), 4 (dotted) GeV^2 (corresponding to $-t = 1.13, 3.30, 5.54 \text{ GeV}^2$ or $\xi = 0.11, 0.12, 0.14$). *Right:* The transition form factor R scaled by $|t|$ in the quark-diquark model versus $|t|$ [GeV^2] for Mandelstam $s = 30 \text{ GeV}^2$.

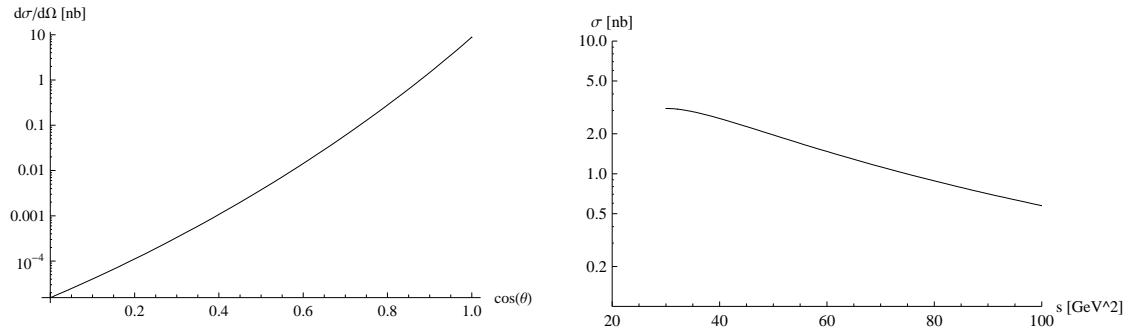


Figure 3: *Left:* The $p\bar{p} \rightarrow \Lambda_c \bar{\Lambda}_c$ differential cross section $d\sigma/d\Omega$ versus $\cos \theta_{\text{cm}}$ at Mandelstam $s = 30 \text{ GeV}^2$. *Right:* The $p\bar{p} \rightarrow \Lambda_c \bar{\Lambda}_c$ integrated cross section σ versus Mandelstam s .

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